Chapter- Zero
(Mathematical Tools)
Topic – 1 (Algebra)

1. <u>Basic Formulae</u>

(i) $(a+b)^2 = a^2 + b^2 + 2\,ab$

(ii) $(a-b)^2 = a^2 + b^2 - 2\,ab$

(iii) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(iv) $(a+b)(a-b) = a^2 - b^2$

(v) $(a+b)^3 = a^3 + b^3 + 3ab\,(a+b)$

(vi) $(a-b)^3 = a^3 - b^3 - 3\,ab\,(a-b)$

(vii) $(a+b)^2 - (a-b)^2 = 4\,ab$

(viii) $(a+b)^2 + (a-b)^2 = 2\,(a^2 + b^2)$

<u>Identity1</u>. $(a+b)^2 = a^2 + b^2 + 2ab$

<u>Proof</u> :-

L. H.S = $(a+b)^2 = (a+b)(a+b)$

$= a^2 + ab + ba + b^2$

$= a^2 + 2ab + b^2$

= R. H.S

<u>Confirmation:</u> If a = 3, b= 2 then

L.H.S = $(a+b)^2 = (3+2)^2 = (5)^2 = 25$

R .H.S = $a^2 + b^2 + 2ab$

$(3)^2 + (2)^2 + 2 \times 3 \times 2$

= 9 + 4 + 12 = 25

From above

L.H.S =R.H.S

2. <u>Quadratic Equation:</u> An equation of second degree is called a quadratic equation.
The standard form of equation is
$ax^2 + bx + c = 0$,

Then $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

<u>Ex -1</u>

$1x^2 + 1x - 2 = 0$, Find value of x

$1x^2 + 1x + (-2) = 0$

a = 1, b =1, c = -2

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-2)}}{2.1}$$

$$= \frac{-1 \pm \sqrt{1+8}}{2} = \frac{-1 \pm 3}{2} = \frac{-4}{2}, \frac{2}{2} = -2, 1$$

<u>Ex -2</u>

$1x^2 + 4x - 5 = 0$, find value of x

$1x^2 + 4x + (-5) = 0$

$a = 1$, b = 4, c =-5

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4\,(1)(-5)}}{2 \times 1}$$

$$= \frac{-4 \pm \sqrt{16+20}}{2 \times 1}$$

$$= \frac{-4 \pm \sqrt{36}}{2 \times 1} = \frac{-4 + 6}{2}, \frac{-4 - 6}{2}$$

$$\left(\frac{2}{2}, \frac{-10}{2}\right) = (1, -5)$$

<u>Ex -3</u>

$10x^2 - 27x + 5 = 0$, find value of x

$10x^2 + (-27)\,x + 5 = 0$

a = 10, b = -27, c =5

$$x = \frac{-(-27) \pm \sqrt{(-27)^2 - 4 \times 10 \times 5}}{2 \times 10}$$

$$= \frac{27 \pm \sqrt{729 - 200}}{20} = \frac{27 \pm \sqrt{529}}{20}$$

$$= \frac{27 \pm 23}{20} = \left(\frac{1}{5}, \frac{5}{2}\right)$$

Ex -4

$4x^2 - 4ax + (a^2 - b^2) = 0$

$4x^2 + (-4a)\ x + (a^2 - b^2) = 0$

$a = 4$, b =-4a, c = $(a^2 - b^2)$

$$x = \frac{-(-4a) \pm \sqrt{(-4a)^2 - 4\,(4)\,(a^2 - b^2)}}{2 \times 4}$$

$$x = \frac{4a \pm \sqrt{16a^2 - 16\,(a^2 - b^2)}}{8}$$

$$x = \frac{4a \pm \sqrt{16a^2 - 16a^2 + 16b^2}}{8}$$

$$x = \frac{4a \pm 4b}{8} = \frac{4a + 4b}{8}, \frac{4a - 4b}{8}$$

$$= \frac{4\,(a+b)}{8}, \frac{4\,(a-b)}{8}$$

$$= \left(\frac{a+b}{2}\right), \left(\frac{a-b}{2}\right)$$

Problems for Practice

1. Solve: $6x^2$ – 13x + 6 = 0 $\left[Ans. \frac{3}{2}\ ;\ \frac{2}{3}\right]$

2. Solve: $x^2 + x$ -2 = 0 $[Ans. 1\ ;\ -2]$

3. Solve: $9x^2 + 15x + 4 = 0$ $\left[Ans. \frac{-1}{3}\ ;\ \frac{-4}{3}\right]$

4. Solve: $3x^2 - 8x + 5 = 0$ $\left[Ans. 1\ or\ \frac{5}{3}\right]$

Topic 2 (Binomial Expansion)

Basic Formulae

$$(1+x)^n = 1 + nx \qquad \text{(if } x << 1)$$

Confirmation

Ex 1. $(1 + \cdot 01)^2 \to ?$

Method 1 $(1 + \cdot 01)^2 = (1 \cdot 01)^2 = 1 \cdot 0201$

Method2 $(1 + \cdot 01)^2 \simeq 1 + 2(\cdot 01) = 1 + \cdot 02 = 1 \cdot 02$
(Binomial) $[\because (1+x)^n = 1 + n.x]$

Ex2. $(1 + \cdot 001)^2$

Method1 $(1 + \cdot 001)^2 = (1 \cdot 001)^2 = 1 \cdot 002001$

Method2 $(1 + \cdot 001)^2 \simeq 1 + 2(\cdot 001) = 1 \cdot 002$
(Binomial) $[\because (1+x)^n = 1 + n.x]$

Ex3. $(1001)^{\frac{1}{3}} = (1000 + 1)^{\frac{1}{3}}$

$\left[1000\left(1 + \frac{1}{1000}\right)\right]^{\frac{1}{3}} = 10\,(1 + \cdot 001)^{\frac{1}{3}}$

$[\because (1+x)^n = 1 + nx]$

$= 10\left[1 + \frac{1}{3}(\cdot 001)\right]$

$= 10\,(1 + \cdot 00033)$

$= 10\,(1 \cdot 00033) \quad = 10 \cdot 0033$

Ex4 $(999)^{\frac{1}{3}} = (1000-1)^{\frac{1}{3}}$

$= \left[1000\left(1-\frac{1}{1000}\right)\right]^{\frac{1}{3}}$

$= [1000(1-\cdot 001)]^{\frac{1}{3}}$

$= 10\ [1-(\cdot 001]^{\frac{1}{3}}$

$[\because (\mathbf{1}+\boldsymbol{x})^{\boldsymbol{n}} = \mathbf{1}+\boldsymbol{n}.\boldsymbol{x}]$

$= 10\ (1 - \frac{1}{3} \times \cdot 001)$

$= 10\ [1-(\cdot 00033)]$

$= 10\ (\cdot 99967)$

= 9.9967

Ex5 $\sqrt{26} = (26)^{\frac{1}{2}}$

$= (25+1)^{\frac{1}{2}} = \left[25\left(1+\frac{1}{25}\right)\right]^{\frac{1}{2}}$

$= (25)^{\frac{1}{2}}\left(1+\frac{1}{25}\right)^{\frac{1}{2}}$

$= 5\ (1+\cdot 04)^{\frac{1}{2}}$

$= 5\left(1+\frac{1}{2}\times \cdot 04\right)$

$[\because (\mathbf{1}+\boldsymbol{x})^{\boldsymbol{n}} = \mathbf{1}+\boldsymbol{nx}]$

= 5 (1 + ·02)

$= \boxed{5 \cdot 10}$

Ex6 $g' = g\frac{R^2}{(R+h)^2}$

$g' = g\frac{R^2}{\left[R\left(1+\frac{h}{R}\right)^2\right]}$

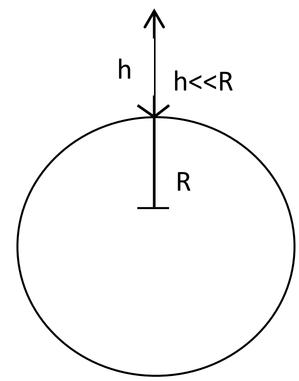
$g' = g.\ \frac{1}{\left(1+\frac{h}{R}\right)^2}$

$g' = g.\left(1+\frac{h}{R}\right)^{-2}$

$g' = g\left(1\ +\ (-2)\ \frac{h}{R}\right)$

$[\because (\mathbf{1}+\boldsymbol{x})^{\boldsymbol{n}} = \mathbf{1}+\boldsymbol{n}.\boldsymbol{x}]$

$$\boxed{g' = g\left(1-\frac{2h}{R}\right)}$$



Problems for Practice

(i) Expand $(33)^{\frac{1}{5}}$ $[Ans: 2.01]$

(ii) Expand $(1.006)^{\frac{-3}{5}}$ $[Ans:\ 0.099]$

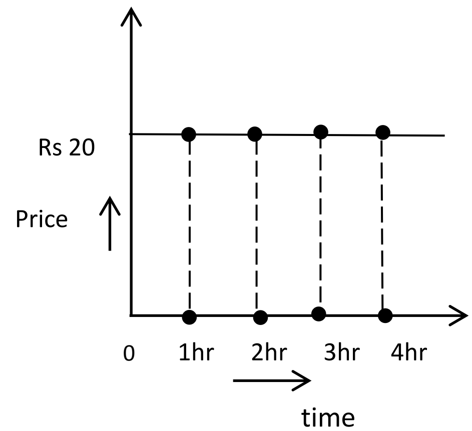
(iii) Expand $\frac{1}{1+x}$, Where (x << 1) $[Ans: (1-x)]$

(iv) Expand $\sqrt{65}$ $[Ans:\ 8.06]$

Topic 3

Graphs

A graph is a line, straight or curved which shows the variation of one quantity w.r.t other, which are interrelated with each other.



Type – 1:

$$y = \text{constant}$$

Ex 1: Price = 20 Rs, It does not depend on time.

$$y = 20$$

$$\frac{Change\ in\ Price}{Change\ in\ time} = \frac{\Delta P}{\Delta t} = \frac{0}{2hr - 1hr} = \frac{0}{1hr} = 0\,\frac{Rs}{hr}$$

$$\frac{\Delta P}{\Delta t} = 0\,\frac{Rs}{hr}$$

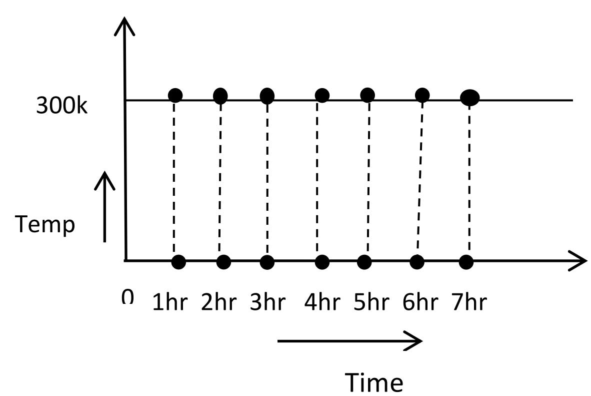
We can write it as $\frac{dP}{dt} = 0\,\frac{Rs}{hr}$

Slope = 0 Rs/hr

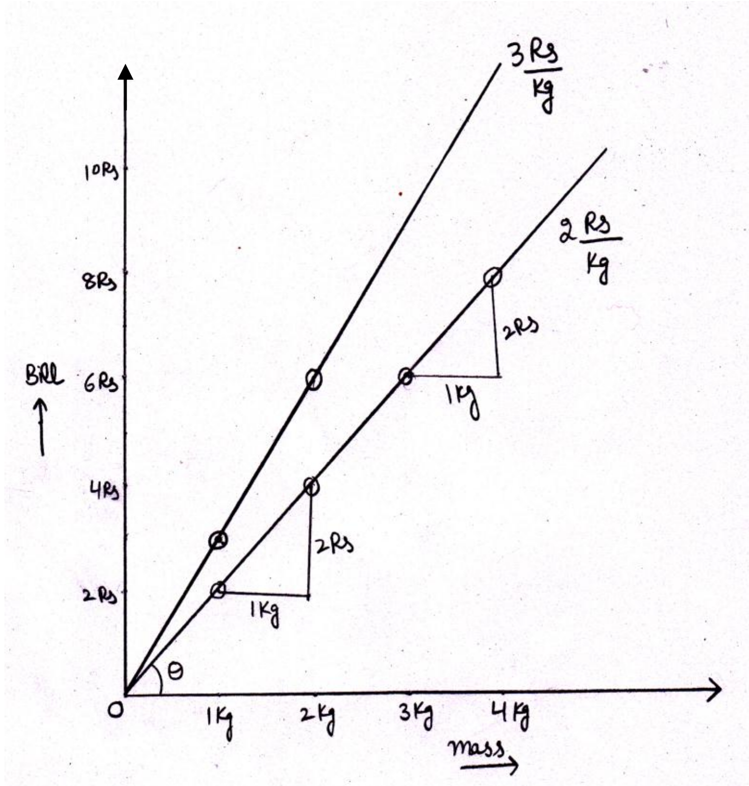
Ex 2: Room Temperature = 27°K (300K)

$$y = 300$$

Condition: We need to maintain the temperature at constant value.



Type –II $y = mx$



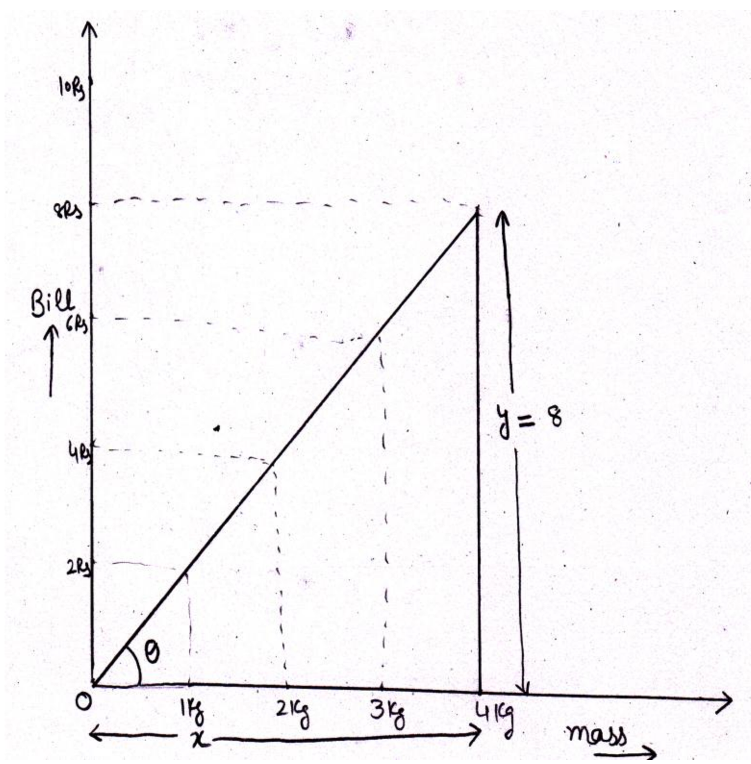
Ex1 Bill = $\left(2\ \frac{Rs}{kg}\right)(m\ kg)$

B = 2 m

$\frac{\Delta B}{\Delta m} = \frac{2Rs}{1kg} = \frac{2Rs}{kg}$

Slope = $\frac{2Rs}{kg} = 2\ \frac{Rs}{kg}$

$$\boxed{\frac{dB}{dm} = 2}$$



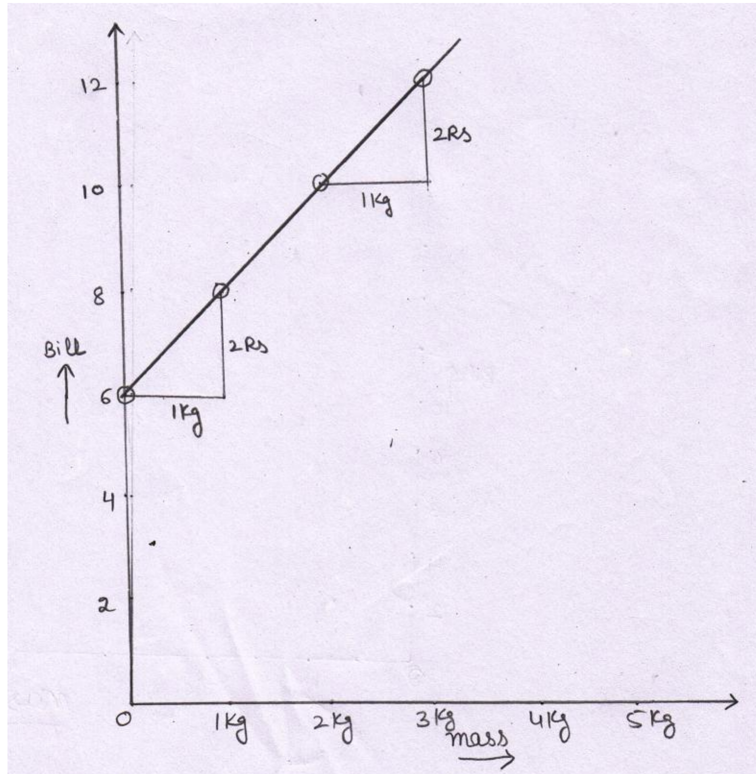
Tan $\theta = \frac{8}{4} = 2\ \frac{Rs}{kg}$

Conclusion

Slope, Tan θ, $\frac{dy}{dx}$ → same

Tan$\theta = \frac{dy}{dx} = 2\ \frac{Rs}{kg}$

Type –(III) $y = mx + c$



Ex -1

Bill = 2 $\left(\frac{Rs}{kg}\right)(m\ \ kg)$ +6Rs

$$\boxed{Bill = 2m + 6}$$

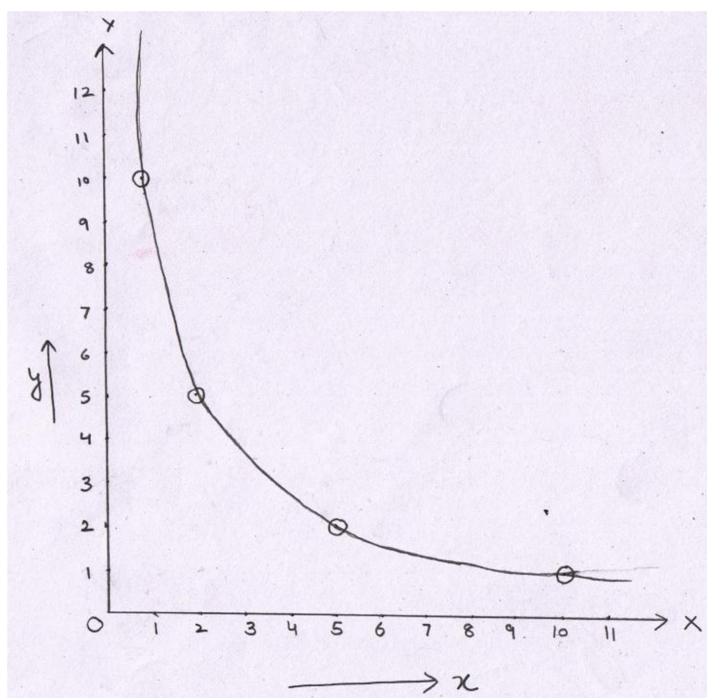
$$\frac{\Delta B}{\Delta m} = \frac{2Rs}{kg} = 2\ \frac{Rs}{kg}$$

Slope, Tan θ, $\frac{dB}{dm} = \ 2\ \frac{Rs}{kg}$

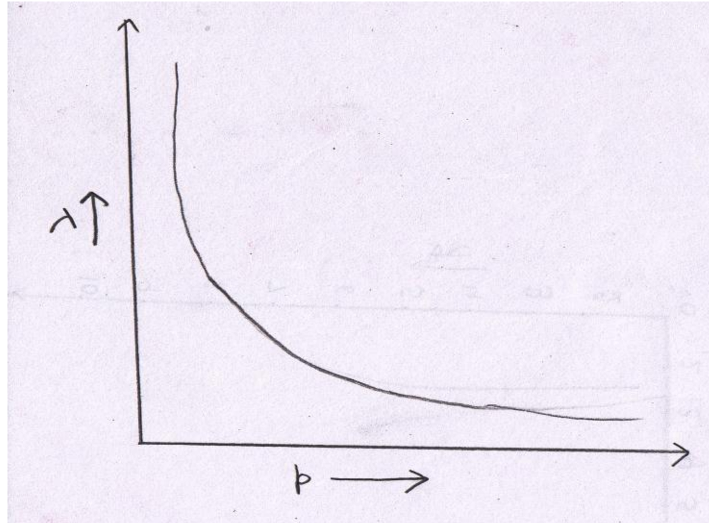
Type (IV)

$y = \frac{10}{x}$

X	Y
1	10
2	5
5	2
10	1



Ex1 $\lambda = \frac{h}{P}$

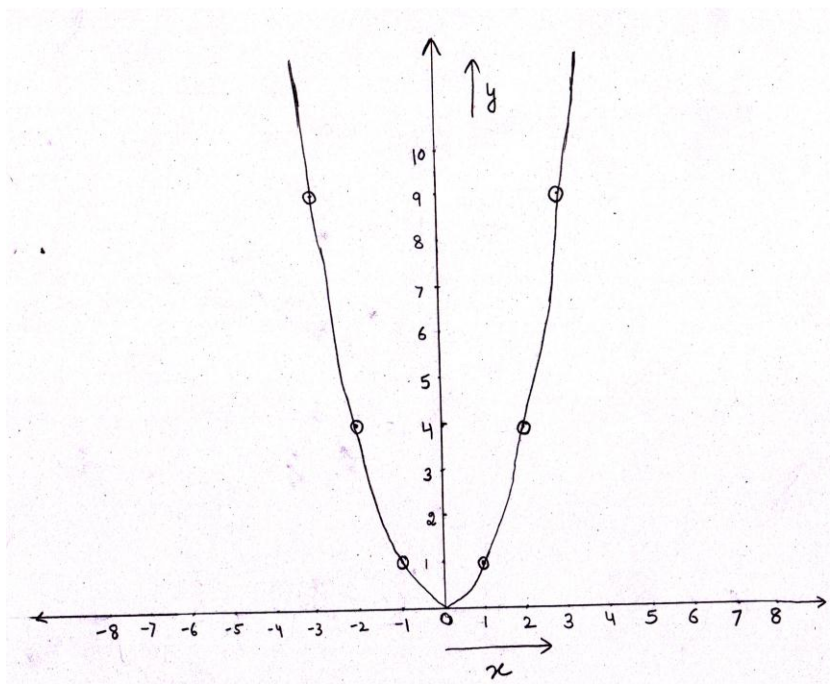


Type V

y = x^2

X	Y
0	0
1	1
2	4
3	9

X	Y
-1	1
-2	4
-3	9



Parabola is symmetrical about y - axis

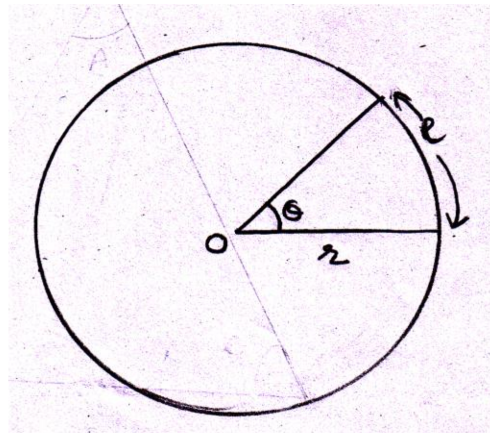
Problem for Practice

Plot the graphs for following.

1. (i) Y = 3, (ii) Y =10
2. (i) Y = 4x, (ii) Y = 6x
3. (i) $Y = 3x + 5$, (ii) $Y = 4x + 6$
4. $Y = \frac{20}{x}$
5. $Y = 2x^2$

Topic-4
Trigonometry

1) Angle $\boxed{\theta = \frac{l}{r}}$



When $l = r$

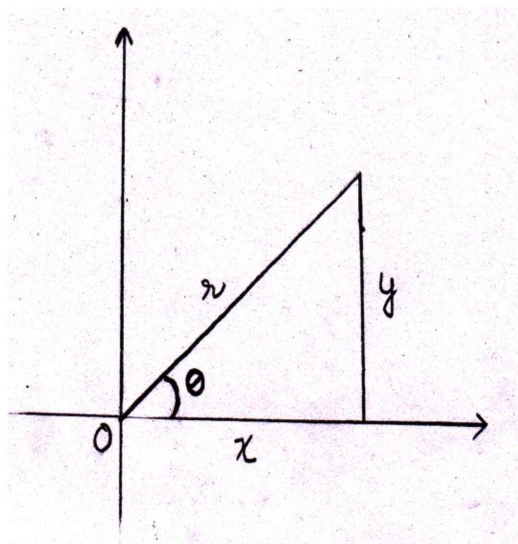
$\theta = \frac{l}{l} = 1$

S.I unit of angle is radian.

2) $\text{Sin}\ \theta = \frac{y}{r} = \frac{\perp ar}{hyp}$

$\text{Cos}\ \theta = \frac{x}{r} = \frac{Base}{hyp}$

$\text{Tan}\ \theta = \frac{y}{x} = \frac{\perp ar}{Base}$



3) **<u>Table</u> (Sinθ, Cosθ, Tanθ)**

	0°	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Sinθ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0
Cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$\frac{-1}{2}$	$\frac{-1}{\sqrt{2}}$	$\frac{-\sqrt{3}}{2}$	-1	0	1
Tanθ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	-$\sqrt{3}$	-1	$\frac{-1}{\sqrt{3}}$	0	$-\infty$	0

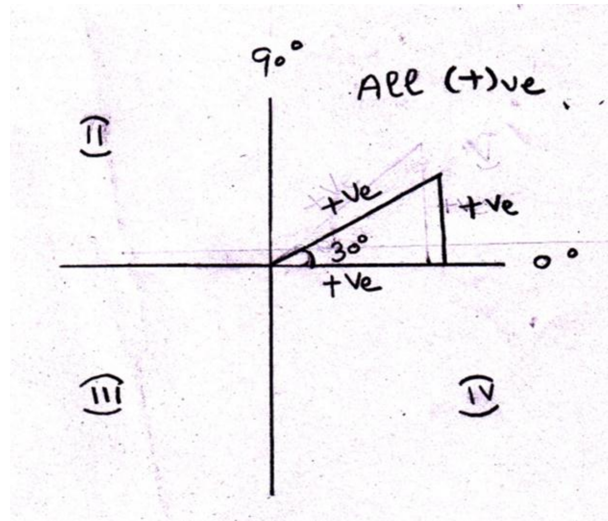
4) Sign in Four Quadrants.

I[st] Quadrant:

Sin 30° = (+)ve

Cos 30° = (+)ve

Tan 30° = (+) ve



2[nd] Quadrant:

Sin 150° = (+)ve

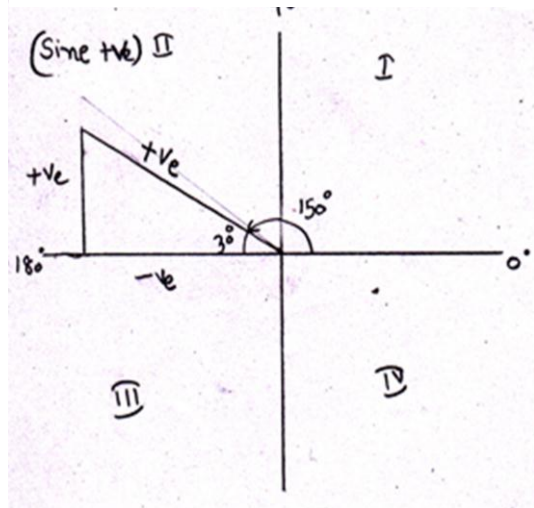
Cos 150° = (-)ve

Tan 150° = (-)ve

Sin 150° = $+\frac{1}{2}$

Cos 150° =$-\frac{\sqrt{3}}{2}$

Tan 150° = $-\frac{1}{\sqrt{3}}$

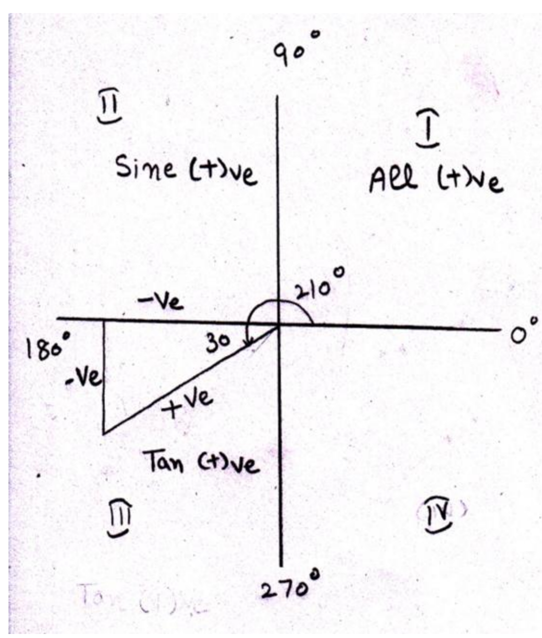


3[rd] Quadrant:

Sin 210° = $-\left(\frac{1}{2}\right)$

Cos 210° = $-\left(\frac{\sqrt{3}}{2}\right)$

Tan 210° = $+\left(\frac{1}{\sqrt{3}}\right)$

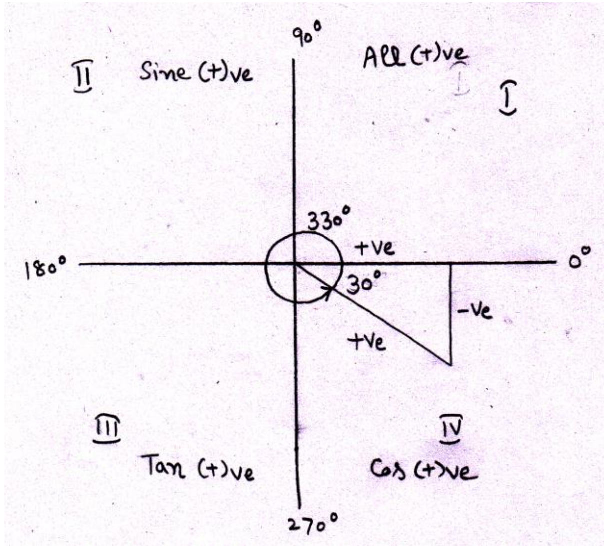


4^{th} Quadrant:

$\text{Sin } 330° = -\left(\frac{1}{2}\right)$

$\text{Cos } 330° = +\left(\frac{\sqrt{3}}{2}\right)$

$\text{Tan } 330° = -\left(\frac{1}{\sqrt{3}}\right)$



5) I) **Sin (A +B) = Sin A. Cos B+ cos A Sin B**

Confirm:

A = 60°, B = 30°

L. H.S	**R. H.S**
Sin (60 + 30) = Sin 90°	= Sin 60° Cos 30° + cos 60° Sin30°
=1	$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2}$
	$\frac{3}{4} + \frac{1}{4}$
	= 1

L.H.S = R. H. S

II) **Cos (A+B) = Cos A Cos B – Sin A Sin B**

Confirm A = 60°, B = 30°

L. H. S	**R.H.S**
Cos (60 +30) = Cos 90°	Cos60° Cos30° – sin 60° Sin30°
= 0	$= \frac{1}{2} \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{1}{2}$
	= 0

L. H.S = R.H.S

III) Tan (A +B) = $\frac{Tan\ A+Tan\ B}{1-Tan\ A\ Tan\ B}$

Confirm:

Let A= 30°, B =30°

L.H.S	**R.H.S**
Tan (30° + 30°) = Tan60°	$\frac{Tan\ 30°+Tan\ 30°}{1-Tan\ 30°\ Tan\ 30°}$
= $\sqrt{3}$	$\frac{\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}.\frac{1}{\sqrt{3}}}$ = $\frac{2\frac{1}{\sqrt{3}}}{1-\frac{1}{3}}$
	$\frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$ = $\sqrt{3}$

$(L.H.S = R.H.S)$

IV) Sin (2A) = Sin (A +A) = Sin A Cos A + Cos A Sin A

Sin 2 A= 2 Sin A Cos A

V) Cos(2A) = Cos (A +A) = Cos A Cos A – Sin A Sin A

= $\text{Cos}^2 A$ - $\text{Sin}^2 A$

= (1 -$\text{Sin}^2 A$) - $\text{Sin}^2 A$

$$\text{Cos(2A)} = 1 - 2\ \text{Sin}^2 A = 2\text{Cos}^2 A - 1$$

6) **Subtraction Formulas:**

1) **Sin (A –B) = Sin A Cos B – Cos A Sin B**

Confirm:

A = 60° , B = 30°

L.H.S	**R .H.S**
Sin (60-30) = Sin 30°	Sin 60° Cos 30° - Cos 60° Sin 30°
$= \frac{1}{2}$	$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{1}{2}$
	$= \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

L.HS = R.H.S

2) **Cos (A –B) = Cos A Cos B + Sin A Sin B**

Confirm: A= 60°, B = 30°

L.H.S	**R.H.S**
Cos (60 -30°) = Cos 30°	Cos 60° Cos 30° + Sin 60° Sin 30°
$= \frac{\sqrt{3}}{2}$	$\frac{1}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{1}{2}$
	$\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

L.H.S = R.H.S

3) $$\boxed{\text{Tan (A-B)} = \frac{Tan\, A - Tan B}{1 + Tan\, A\, Tan\, B}}$$

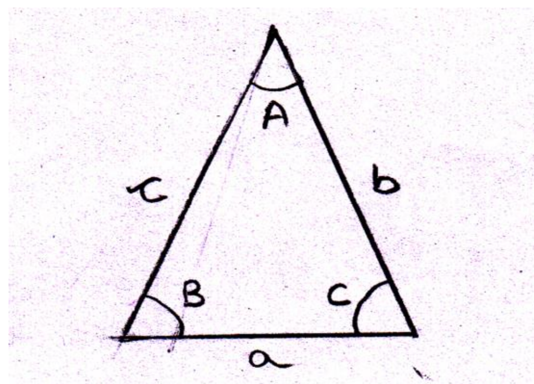
A = 60°, B = 30°

L.H.S	**R.H.S**
Tan (60° -30°) = Tan 30°	$\frac{Tan\ 60° - Tan\ 30°}{1 + Tan\ 60°\ Tan\ 30°}$
$\frac{1}{\sqrt{3}}$	$\frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}} = \frac{\frac{3-1}{\sqrt{3}}}{1+1} = \frac{\frac{2}{\sqrt{3}}}{2}$
	$= \frac{2}{\sqrt{3}} \times \frac{1}{2} = \frac{1}{\sqrt{3}}$

L.H.S = R.H.S

7) **<u>Triangle</u>**

1) $$\boxed{\frac{a}{Sin\ A} = \frac{b}{Sin\ B} = \frac{c}{Sin\ C}}$$

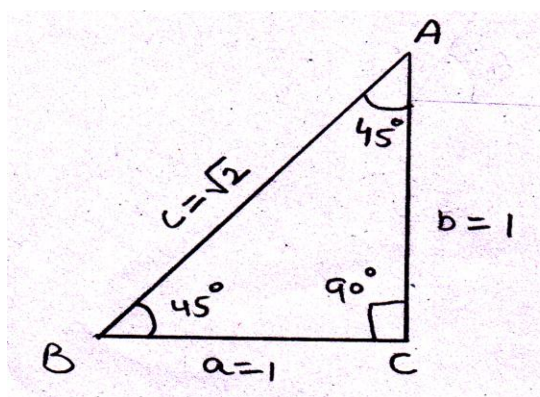


<u>Confirm:</u>

$$\frac{1}{Sin\ 45°} = \frac{1}{Sin\ 45°} = \frac{\sqrt{2}}{Sin\ 90°}$$

$$\frac{1}{\frac{1}{\sqrt{2}}} = \frac{1}{\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}}{1}$$

$$\boxed{\sqrt{2} = \sqrt{2} = \sqrt{2}}$$



2) $$\boxed{\text{CosA} = \frac{b^2 + c^2 - a^2}{2bc}}$$

Problems for Practice

1. Find the value of cos $(120)^{\circ}$, Sin $(120)^{\circ}$ Tan $(120)^{\circ}$.

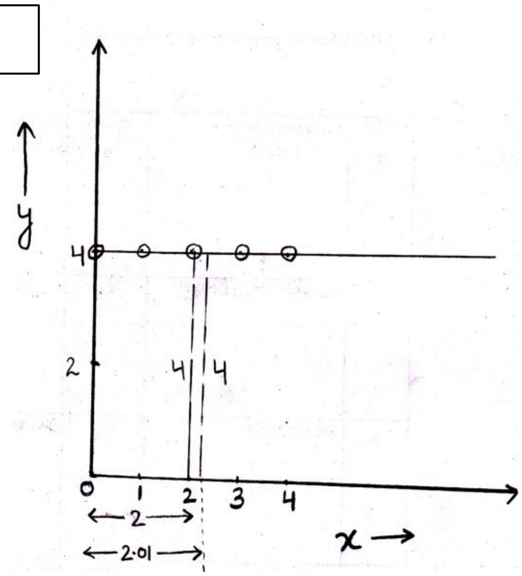
$\left[\boldsymbol{Ans.} \left(\frac{-1}{2}\right), \frac{\sqrt{3}}{2}, -\sqrt{3}\right]$

2. Convert 60° into radians. **[Ans, $\frac{\pi}{3}$ radians]**

3. Convert 0.6 radian into degree. **[Ans. 34. 38°]**

Topic-5
Differentiation

Differentiation (Slope) = Tanθ



Type-1: y = Constant

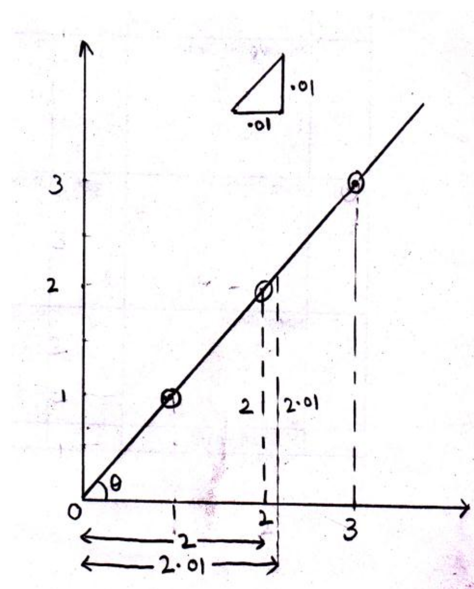
a) Slope =0

b) Tanθ = Tan 0 = 0

c) $\frac{dy}{dx} = \frac{\Delta y}{(\Delta x)_{lt\ \Delta x \to 0}} = \frac{4-4}{2.01-2.00}$

$= \frac{0}{\cdot 01}$

$\frac{dy}{dx} = 0$



Type-2: $y = x$

a) Slope = Tanθ = $\frac{3}{3}$ = 1

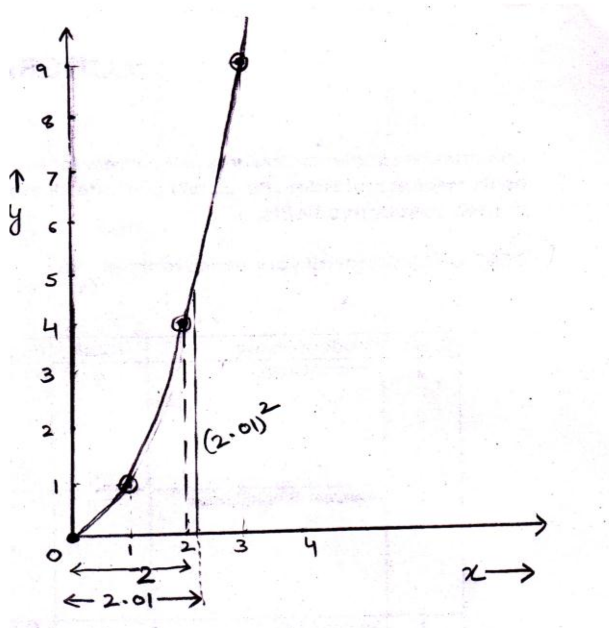
b) $\frac{dy}{dx} = \frac{\Delta y}{(\Delta x)_{lt\ \Delta x \to 0}} = \frac{2.01-2.00}{2.01-2.00}$

$= \frac{\cdot 01}{\cdot 01} = 1$

$\frac{dy}{dx} = 1$

Type-3:

Ex-1: $y = x^2$



$$\frac{dy}{dx} = \frac{\Delta y}{\Delta x_{lt\ \Delta x \to 0}} = \frac{(2.01)^2 - 2^2}{2.01 - 2.00}$$

$$= \frac{(2 + \cdot 01)^2 - 2^2}{2 \cdot 01 - 2 \cdot 00}$$

$$= \frac{2^2 + (\cdot 01)^2 + 2 \times 2\ (\cdot 01) - 2^2}{\cdot 01}$$

$$= \cdot 01 + 4$$

$$\approx 4$$

In general form

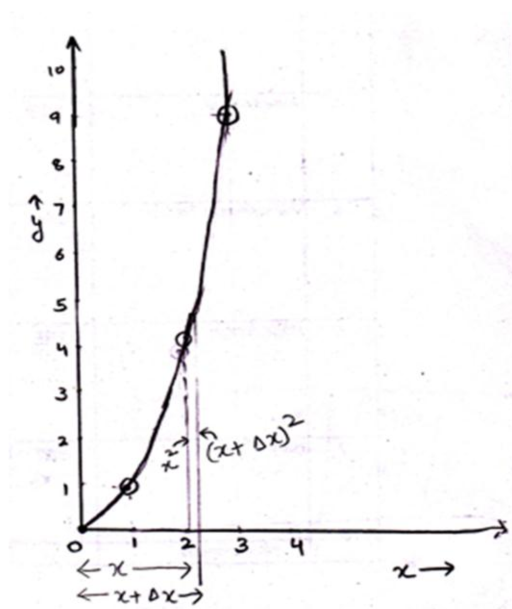
$$\frac{dy}{dx} = \frac{\Delta y}{\Delta x_{lt\ \Delta x \to 0}} = \frac{(x + \Delta x)^2 - x^2}{x + \Delta x - x}$$

$$= \frac{x^2 + (\Delta x)^2 + 2x\ \Delta x - x^2}{\Delta x}$$

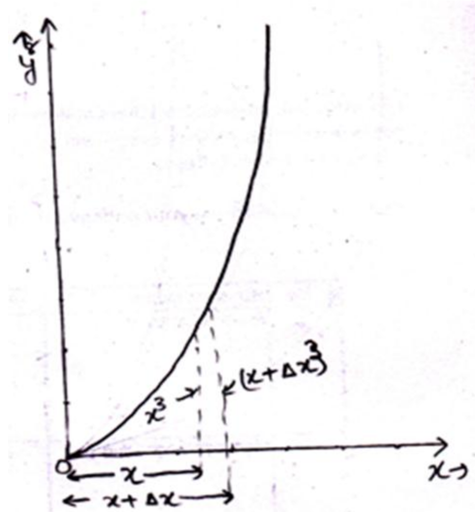
$$= \Delta x + 2x$$

$$\boxed{\frac{dy}{dx} = 2x}$$

As Δx is very small so can be neglected



Ex 2: $y = x^3$



$$\frac{dy}{dx} = \frac{\Delta y}{\Delta x_{\,lt\ \Delta x \to 0}} = \frac{(x+\Delta x)^3 - x^3}{x+\Delta x - x}$$

$$= \frac{x^3 + (\Delta x)^3 + 3 \times x \times \Delta x (x+\Delta x) - x^3}{\Delta x}$$

$$= \Delta x^2 + 3 \times x \ \times (x + \Delta x)$$

$$= 3x^2 + 3x\,\Delta x + \ (\Delta x)^2$$

$$= 3x^2 \quad [\because 3x\,\Delta x, \Delta x^2 \ are\ very\ small\ so\ neglected]$$

$$\frac{dy}{dx} = 3x^2$$

Basic Formula

If $y = x^n$

Then, $\boldsymbol{\frac{dy}{dx} = nx^{n-1}}$

Product rule

<u>**Type-4**</u>:

$$y = u.v$$

$$\frac{dy}{dx} = \frac{d}{dx}(u.v)$$

$$= u\frac{dv}{dx} + v\frac{du}{dx}$$

Ex1: $y = (2x^3 + 3)\ (2x^{-3} + 1)$

$$\frac{dy}{dx} = (2x^3 + 3)\frac{d}{dx}(2x^{-3} + 1) + (2x^{-3} + 1)\frac{d}{dx}(2x^3 + 3)$$

$$\frac{dy}{dx} = (2x^3 + 3)\ [(2(-3)x^{-4} + 0)] + (2x^{-3} + 1)[(2(3)x^2 + 0)]$$

$$\frac{dy}{dx} = (2x^3 + 3)\ (-6x^{-4}) + (2x^{-3} + 1)\ (6x^2)$$

$$\boxed{\frac{dy}{dx} = 6x^2 - 18x^{-4}}$$

Type5:

$$y = (ax + b)^n$$

$$\frac{dy}{dx} = \frac{d}{dx}[(ax + b)^n]$$

$$= n(ax + b)^{n-1}.\ \frac{d}{dx}(ax + b)$$

Ex1: $y = (2x + 3)^4$

$$\frac{dy}{dx} = \frac{d}{dx}[(2x + 3)^4]$$

$$= 4(2x + 3)^3.\frac{d}{dx}(2x + 3)$$

$$= 4(2x + 3)^3.\ (2 \times 1 + 0)$$

$$= 8\ (2x + 3)^3$$

Type6:

y = Sinθ

$$\frac{dy}{dx} = \frac{d}{dx}\,[\text{Sin}\theta]$$

$$= \left[Cos\theta\ \frac{d(\theta)}{dx}\right]$$

Ex1: $y\ = \text{Sin}x$

$$\frac{dy}{dx} = \text{Cos}\ x\left(\frac{dx}{dx}\right)$$

$$= \text{Cos}\ x.1$$

$$= \text{Cos}\ x$$

Ex2: $y\ = \text{Sin}\ (2x)$

$$\frac{dy}{dx} = \frac{d}{dx}\ (\text{Sin}2x)$$

$$= \text{Cos}2x\ \frac{d}{dx}\ (2x)$$

$$= \text{Cos}\ (2x)\,.\,(2)$$

$$=\ 2\ \text{Cos}2x$$

Type7: $y\ = \text{Cos}\theta$

$$\frac{dy}{dx} = \frac{d}{dx}\ (\text{Cos}\theta)$$

$$= -\ \text{Sin}\theta\ .\ \frac{d\theta}{dx}$$

Ex1: $y = \text{Cos}x$

$$\frac{dy}{dx} = -Sin\ x\ \frac{dx}{dx}$$

$$\boxed{\frac{dy}{dx} = -Sinx}$$

Ex2: y = Cos $(2x)$

$\frac{dy}{dx} = \frac{d}{dx}(\text{Cos}2x)$

$= -\text{ Sin } 2x \frac{d}{dx}(2x)$

$= -\text{Sin } 2x.\ (2)$

$= -2 \text{ Sin } 2x$

Ex3: $\frac{d}{dx}(logx) = \frac{1}{x}$

Ex4: $\frac{d}{dx}(e^x) = e^x$

Problems for Practice:

1. Given V = $\frac{4}{3}\pi r^3$, find $\frac{dv}{dr}$ [Ans: 4 πr^2]

2. Evaluate the derivative of function $y = 3x^{-3}\ at\ x = 3$

3. Differentiate the following $w.r.t\ x$

(i) $\sqrt{x} - \frac{1}{\sqrt{x}}$ $\left[\text{Ans: } \frac{1}{2\sqrt{x}}\ \left(\frac{x+1}{x}\right)\right]$

4. Differentiate the following w.r.t x

(i) $(5x^2+6)\ (2x^3+4)$ $[\text{Ans.}\ 50x^4 + 36x^2 + 40x]$

5. If the displacement x of a particle (in metre) is related with time (in second) according to relation

$$x = 2t^3 - 3t^2 + 2t + 2$$

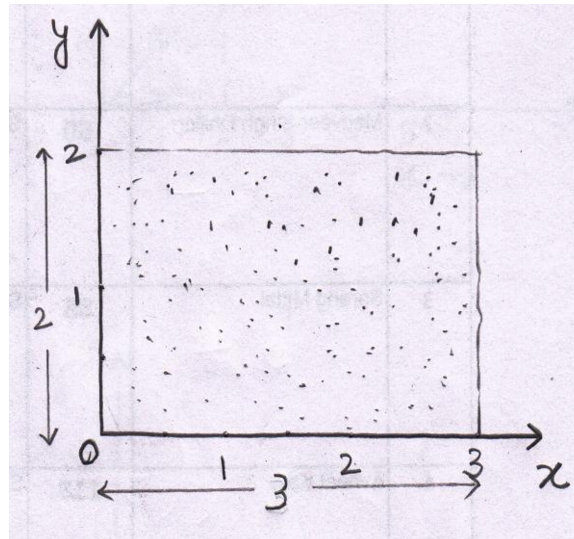
Find the position, velocity and acceleration of a particle at the end of 2 seconds.
[Ans. 10 m, 14m/s, 18m/s^2]

Topic-6
Integration

The Process of integration is just the reverse of differentiation. The symbol used for integration is $\int$. In physics we use it to find area under graph.

Ex-1: If y = 2, find area under the graph.

Area = 2 × 3 = 6

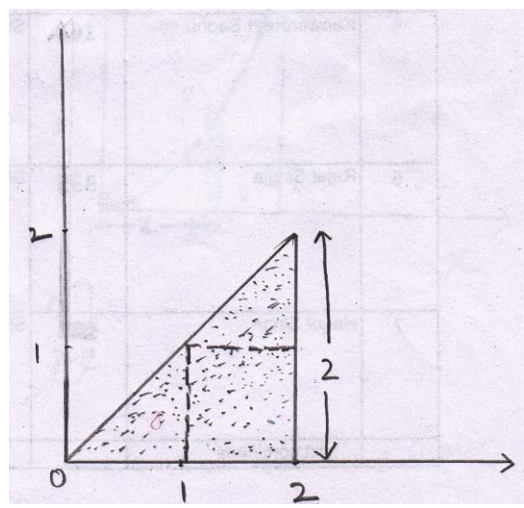


Ex-2: Area = ?

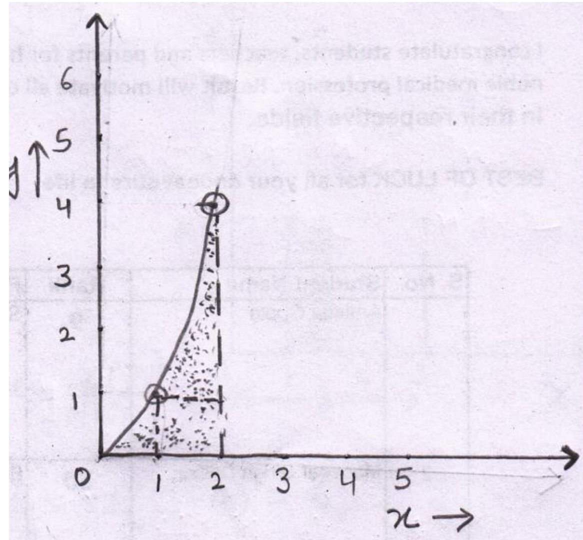
$= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 2 \times 2$

= 2 $(ft)^2$

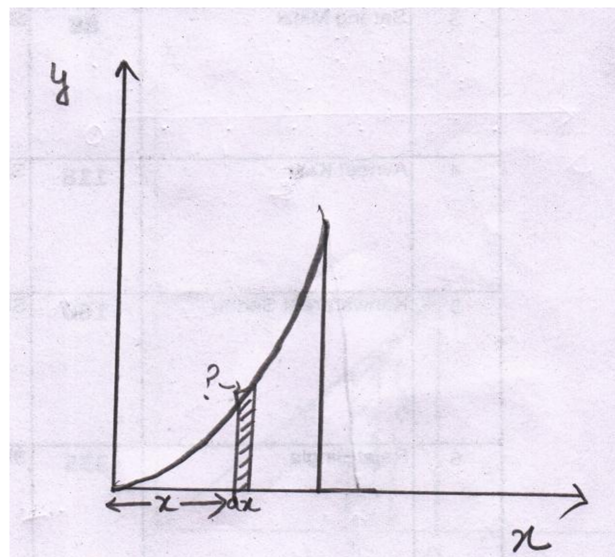


Ex-3: $y = x^2$



X	Y
1	1
2	4

Small Area, $dA = y.\, dx$



Total Area

$= \sum^{\infty} y\, dx$

$= \int y \;\; dx$

$= \int x^2\, dx$

$= \left|\frac{x^{2+1}}{2+1}\right| = \left|\frac{x^3}{3}\right|_0^2$

$\left[\frac{2^3}{3} - \frac{0^3}{3}\right] = \left[\frac{8}{3} - 0\right]$

$= \frac{8}{3} = 2.67$

Ex-1: By method of integration

Area = $\int y\, dx$

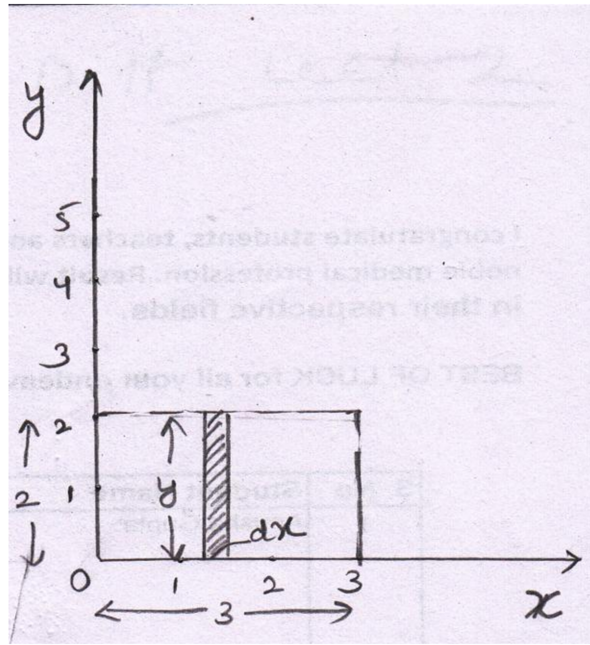
= $\int 2\, dx$

= $2\int 1\, dx$

= $2\int x^0\, dx \left[\int x^n dx = \frac{x^{n+1}}{n+1}\right]$

= $2\left|\frac{x^{0+1}}{0+1}\right|_0^3$

= $2\,|x|_0^3$ = 2 (3 -0) =6



Ex-2: $y = x$

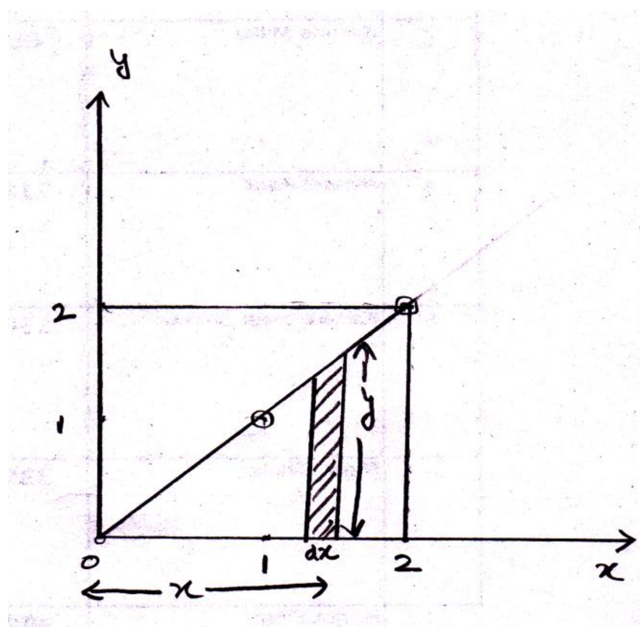
$dA = y.dx$

$A = \int ydx$

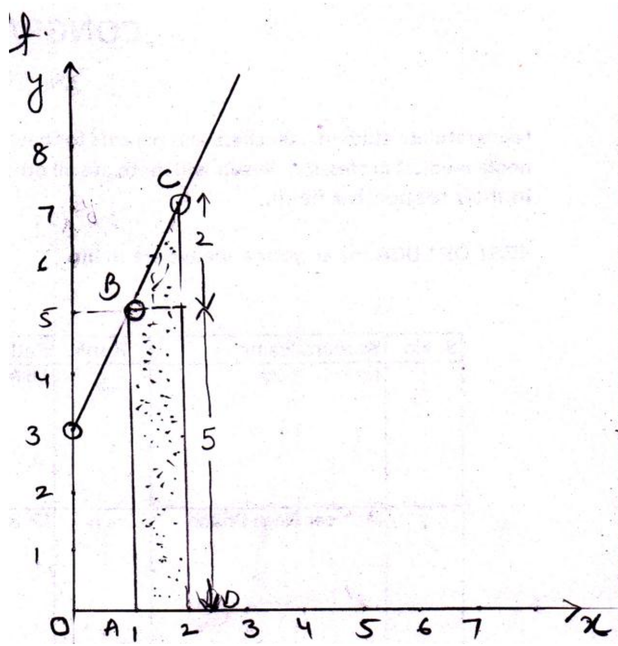
= $\int xdx$

= $\left|\frac{x^{1+1}}{1+1}\right|_0^2$ = $\left|\frac{x^2}{2}\right|_0^2$

= $\left|\frac{2^2}{2} - \frac{0^2}{2}\right|$ = (2 -0) = 2



Ex-3: **Try yourself .** $y = x^2$.



Ex-4:

$$\boxed{y = 2x + 3}$$

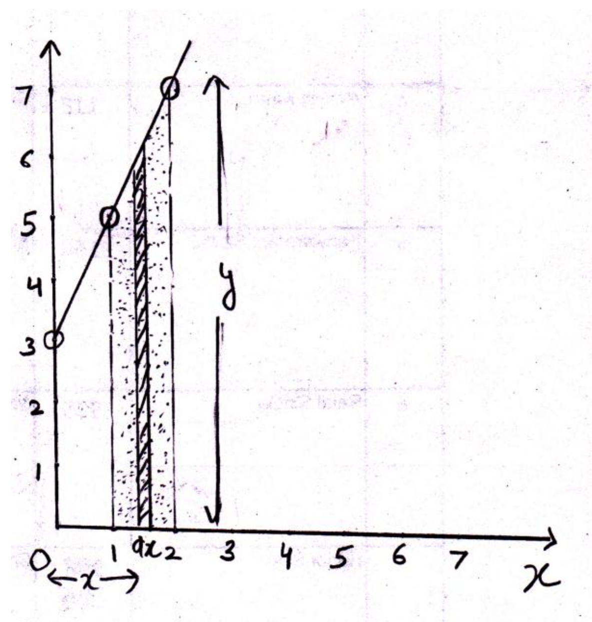
Find Area ABCD

Method 01:

$=1 \times 5 + \frac{1}{2} \times 1 \times 2$

$= 5 + 1 = 6$

Method02: $= \int y dx$



$= \int (2x + 3).dx$

$= 2 \int x^1 dx + 3 \int x^0 dx$

$= 2 \left[\frac{x^2}{2}\right]_1^2 + 3 \left[\frac{x^1}{1}\right]^2$

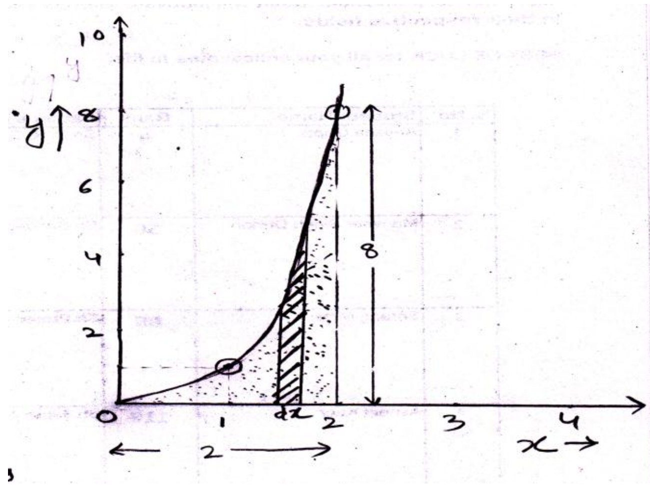
$= [x^2]_1^2 + 3[x]_1^2$

$= [2^2 - 1^2] + 3\,[2 - 1]$

$= [4 - 1] + 3\,(1)$

$= 3 + 3 = 6$

Ex5: $y = x^3$ where, $0 \le x \le 2$



Area = $\int y dx$

= $\int x^3 dx$

= $\left|\frac{x^{3+1}}{3+1}\right|_0^2$

= $\left|\frac{x^4}{4}\right|_0^2$

= $\left[\frac{2^4}{4} - \frac{0^4}{4}\right]$ = 4

Integration formula's

i) $\int x^n \ dx = \frac{x^{n+1}}{n+1}$ $\left[\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = \frac{(n+1)\,x^{n+1-1}}{(x+1)} = x^n\right]$ *Confirm*

ii) $\int x^0 . dx = \frac{x^{0+1}}{0+1} = x$

iii) $\int (u + v) dx = \int u dx + \int v \ dx$

iv) $\int \frac{1}{x}$ .dx = log_cx

v) $\int sinx \ dx = -\cos x$

vi) $\int cosx \ dx = \sin x$

vii) $\int (ax + b)^n \ dx = \frac{(ax+b)^{n+1}}{(n+1)a}$

Problems for Practice

1. Integrate $2\,x^{\frac{-2}{5}}$ $\left[Ans. \frac{10}{3}\ x^{3/5}\right]$

2. Integrate $\frac{1}{1+x}$ $[Ans.\, ln\ (x+1)]$

3. Integrate $\frac{3}{2}\ x^5 + \frac{3}{x^2}$ $\left[Ans. \frac{1}{4}\ x^6 - 3\,\frac{1}{x}\right]$

4. Integrate $\left(1\ + \frac{1}{x} - \frac{1}{x^2}\right)$ $\left[Ans.\, x + lnx + \frac{1}{x}\right]$

5. Integrate $\frac{1}{2}\sqrt{x}$ $\left[Ans. \frac{1}{3}\ x^{3/2}\right]$

6. Evaluate $\int_0^{30} \cos 5x dx$ $\left[Ans. \frac{1}{10}\right]$

7. Evaluate $\int e^{kx}\ dx$ $\left[Ans. \frac{e^{kx}}{k}\right]$

8. Evaluate $\int_0^{\pi/2} (1+sinx)^{1/2}\ dx$ $[Ans.\, 2]$

9. $\int_R^{\infty} \frac{GMm}{x^2}\ dx$ $\left[Ans. \frac{GMm}{R}\right]$

10. Evaluate $\int \left(\frac{1}{ax+b}\right)\ dx$ $\left[Ans. \frac{1}{a}\ log_e\ (ax+b)\right]$

11. $\int_0^{\pi/4} Sinx \cos x dx$ $\left[Ans. \frac{1}{4}\right]$

Topic-7

Logarithm

7.1 $log_a N = x$ N = a^x

Ex1: log_{10} 100 = 2 (100 = 10^2)

Ex2: log_5 25 = 2

Ex3: log_5 625 = 4

Ex4: log_3 27 = 3

7.2 Basic formulae of Logarithm

i) $log_a mn = log_a m + log_a n$

ii) $log_a \frac{m}{n} = log_a m - log_a n$

iii) $log_a m^n = n \, log_a m$

iv) $log_a m = log_b m \times log_a b$

7.3 $log_e m$ = 2.3 $log_{10} m$ (e = 2.718)